HEAT AND VAPOR TRANSPORT IN ICING-UP ON A HEAT EXCHANGER

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UDC 536.42

An approximate equation has been derived for the thickness of an ice layer during growth, and a comparison is made with experiment.

Heat calculations and heat exchangers for agricultural buildings require determination of the thickness of the layer of ice resulting from condensation of water vapor [1, 2]; the thickness of this layer varies with the outside temperature.

Here we solve the equations of thermal conduction with a moving phase-transition boundary to determine the thickness as a function of time (we neglect the thermal resistance of the metal wall involved in the condensation).

This ice grows by condensation of water vapor (t_1 = 283°K, relative humidity $\varphi \approx 80\%$), which subsequently freezes; this results in a two-layer medium (water–ice), and the surface of the water layer has the dew-point temperature, which corresponds to the above parameters of the inside air. However, to simplify the treatment we assume that the layer of water is infinitely thin, i.e., the latent heat of condensation and that of solidification are released at the phase-transition boundary (we neglect the heat transfer between the internal air and the surface of the water at the dew-point temperature).

The mathematical problem is to solve the thermal-conduction equation with the appropriate initial and boundary conditions (the x axis is perpendicular to the heat-transfer surface, x = 0, and directed in the sense of growth of the ice layer):

$$\rho c \frac{\partial t}{\partial \tau} = \frac{\partial^2 t}{\partial x^2}, \tag{1}$$

$$t|_{\tau=0} = 0, \tag{2}$$

$$t|_{x=\xi} = 0, \tag{3}$$

$$\lambda \frac{\partial t}{\partial x}\Big|_{x=\xi} = \rho (r + r_1) \frac{d\xi}{d\tau}, \tag{4}$$

$$-\lambda \frac{\partial t}{\partial x}\Big|_{x=0} = \alpha_2 (t_2 - t|_{x=0}). \tag{5}$$

The temperature t_2 of the outside air varies as follows [3]:

$$t_2 = t_{av} - b \cos \omega\tau, \tag{6}$$

where b is the amplitude of the fluctuation.

We use an integral method [4, 5] to solve the problem.

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Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 29, No. 3, pp. 479–482, September, 1975. Original article submitted February 18, 1975.

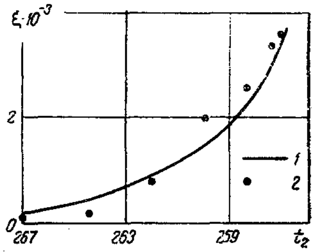

Fig. 1. Thickness ξ (m) of ice layer in relation to outside air temperature t_2 (°K): 1) theory; 2) measurement.

We integrate both sides of (1) with respect to x:

$$\rho c \int_0^\xi \frac{\partial t}{\partial \tau} dx = \lambda \frac{\partial t}{\partial x}\Big|_{x=\xi} - \lambda \frac{\partial t}{\partial x}\Big|_{x=0}$$

or, using (3)-(5) and extracting the derivative with respect to τ from the integral on the left, we have

$$\rho c \frac{d}{d\tau} \int_0^\xi t dx = \rho (r + r_1) \frac{d\xi}{d\tau} + \alpha_2 (t_2 - t|_{x=0}). \quad (7)$$

We integrate (7) with respect to τ:

$$\int_0^\xi t dx - \xi \frac{r + r_1}{c} = \frac{\alpha_2}{\rho c} \int t_2 d\tau - \frac{\alpha_2}{\rho c} \int t|_{x=0} d\tau + B. \quad (8)$$

The functions t and $t|_{x=0}$ in the integral terms in (8) are unknown; the significance of the integral methods in such problems is [4, 5] that they provide a means of choosing a value for $t(x, \tau)$ close to the true one. Then the value of the integral of this function will differ only very slightly from the true value. Since the phase front moves slowly in the present case, we will assume that the temperature distribution differs only slightly from the corresponding distribution for $d\xi/d\tau = 0$, so we substitute the solution to the following problem into the integral terms in (8):

$$\frac{\partial t}{\partial \tau} = a \frac{\partial^2 t}{\partial x^2}, \quad (9)$$

$$t|_{x=\xi} = 0, \quad (10)$$

$$-\lambda \frac{\partial t}{\partial x}\Big|_{x=0} = \alpha_2 (t_2 - t|_{x=0}), \quad (11)$$

$$t|_{\tau=0} = 0. \quad (12)$$

To solve (9)-(12) we perform a Laplace transformation with respect to τ; we use series expansions for the hyperbolic sine and cosine and apply the expansion theorem [6] to obtain the following (since $\xi\sqrt{\omega}/a \ll 1$, and thus the phase shift and amplitude change are practically zero):

$$t \approx \frac{\alpha_2}{(1 + \mathrm{Bi}) \lambda} (t_{av} - b \cos \omega\tau)(\xi - x). \quad (13)$$

We substitute (13) into (8) to get

$$\frac{\alpha_2}{\lambda} \frac{\xi^2}{2(1 + \mathrm{Bi})} (t_{av} - b \cos \omega\tau) - \xi \left[\frac{r}{c} - \frac{\alpha_2}{\rho c} \frac{\alpha_2}{\lambda} \frac{t_{av} - \frac{b}{\omega} \sin \omega\tau}{1 + \mathrm{Bi}} \right] = \frac{\alpha_1}{\rho c} t_1 \tau + \frac{\alpha_2}{\rho c} \left(t_{av}\tau - \frac{b}{\omega} \sin \omega\tau \right) + B. \quad (14)$$

It can be shown that the term quadratic in ξ in (14) is much less than the others, so we have approximately that

$$\xi = - \frac{\frac{\alpha_2}{\rho c} \left[t_{av}(\tau - \tau_0) - \frac{b}{\omega} (\sin \omega\tau - \sin \omega\tau_0) \right] + B}{\frac{r + r_1}{c} \frac{\alpha_2}{\rho c} \frac{\alpha_2 \left[t_{av}(\tau - \tau_0) - \frac{b}{\omega} (\sin \omega\tau - \sin \omega\tau_0) \right]}{\lambda (1 + \mathrm{Bi})}}, \quad (15)$$

where B is defined from the condition $\xi = \delta$ at $\tau = \tau_0$:

$$B = \delta^2 \frac{\alpha_2}{2\lambda (1 + \mathrm{Bi})} (t_{av} - b \cos \omega\tau_0) - \delta \frac{r + r_1}{c} \approx - \delta \frac{r + r_1}{c}. \quad (16)$$

Figure 1 shows calculations from (15) and measurements for the ice-layer thickness as a function of the outside air temperature; we use the following initial values: $\lambda = 2.33$ W/m·deg, $\rho = 900$ kg/m³, $a = 4.45 \cdot 10^{-3}$ m²/hr, c = 2093.4 J/kg·deg, $\alpha_1 = 26.75$ W/m²·deg, $\alpha_2 = 31.4$ W/m²·deg, $t_1 = 283$°K, $t_{av} = 267$°K, $r_1 = 2.257 \cdot 10^6$ J/kg, $r = 3.35 \cdot 10^5$ J/kg, b = 263°K.

In spite of the assumptions and simplifications, the calculations from (15) agree satisfactorily with experiment.

NOTATION

t_2 is the outer air temperature;
t_{av} is the average temperature of outer air;
t_1 is the temperature of inner air;
τ is the time;
Bi is the Biot number;
a is the thermal diffusivity;
δ is the initial thickness of ice layer;
ξ is the coordinate of phase-change boundary;
λ is the thermal conductivity;
r is the latent heat of fusion;
r_1 is the latent heat of evaporation;
c is the specific heat;
ρ is the density;
α_2 is the heat-transfer coefficient;
ω is the frequency of temperature fluctuatios of outside air; T = 24 h, period of temperature fluctuations in outside air.

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